8.5
Applications of Vectors
1. Draw the vectors that represent a person walking 30 meters east, then 40 meters north. Find the magnitude and direction of the resultant vector.

Using the Pythagorean Theorem the resultant vector has a magnitude of 50 meters.

The resultant vector of a person walking shows the displacement of the person, not the total distance traveled. For the above problem, the person walked a distance of 70 meters, but the displacement, or change in position, was only 50 meters.
2. A person walks 22 meters west then 17 meters east.

   a. Find the distance the person traveled.
   \[ 39 \text{ total meters} \]

   b. What is the person’s displacement?
   \[ 5 \text{ meters \ w} \]

   c. Determine the angle of the resultant vector.
3. Example 1, page 513

Two rodeo clowns are restraining a bull. One clown is exerting a force of 270 newtons due north and the other is pulling with a force of 360 newtons due east. Use this information to describe the resultant force on the bull?

a. draw and label a diagram
b. determine the resultant force
c. find the angle the resultant force makes with the east-west axis
In physics, if a constant force $\vec{F}$ displaces an object an amount represented by $\vec{d}$, it does work on the object. The amount of work is given by $W = \vec{F} \cdot \vec{d}$.

4. Example 2, page 514
He is pushing a cart full of packages weighing 100 pounds up a ramp 8 feet long at an incline of 25 degrees. Find the work done by gravity as the cart moves the length of the ramp.
Sometimes there is no motion when several forces are at work on an object. When the forces balance one another, the object is in equilibrium, and the resultant force is zero.

5. Example 3, page 515

Ms. Davis is hanging a sign for her restaurant. The sign is supported by two lightweight support bars as shown in the diagram. If the bars make a 30° angle with each other, and the sign weighs 200 pounds, what are the magnitudes of the forces exerted by the sign on each support bar?

$\vec{F}_1$ represents the force exerted on bar 1 by the sign, $\vec{F}_2$ represents the force exerted on bar 2 by the sign, and $\vec{F}_w$ represents the weight of the sign.
Remember that equal vectors have the same magnitude and direction. So by drawing another vector from the initial point of $\vec{F}_1$ to the terminal point of $\vec{F}_w$, we can use the sine and cosine ratios to determine $|\vec{F}_1|$ and $|\vec{F}_2|$.

\[
\sin 30 = \frac{200}{|\vec{F}_2|} \quad \quad \tan 30 = \frac{200}{|\vec{F}_1|}
\]

$|\vec{F}_2| = \frac{200}{\sin 30} = 400$ lbs.

$|\vec{F}_1| = \frac{200}{\tan 30} \approx 346.4$ lbs.
6. Suppose Len and his tag team wrestling partner are each pulling on the arms of their wrestling opponent. Len exerts a force of 180 lbs due north while his partner exerts a force of 125 pounds due east.

a. Draw a labeled diagram that represents the forces.
b. Determine the resultant force exerted on the opponent.
c. Determine the angle the resultant force makes with the east-west axis.

\[ 180^2 + 125^2 = x^2 \]
\[ x = \sqrt{180^2 + 125^2} = 219.15 \text{ lbs} \]

\[ \tan(\theta) = \frac{180}{125} \]
\[ \theta = \tan^{-1}\left(\frac{180}{125}\right) \approx 55.22^\circ \]
7. Jesse works for a package delivery service. Suppose he is pushing a cart full of packages weighing 125 pounds up a ramp 10 feet long at an angle of 20 degrees. Find the work done by gravity as the cart moves the length of the ramp. Assume that friction is not a factor.

\[
\begin{align*}
\cos(20) &= \frac{x}{10} \\
10\cos(20) &= x \\
\sin(20) &= \frac{y}{10} \\
10\sin(20) &= y
\end{align*}
\]

\[\vec{d} = \langle 10\cos(20), 10\sin(20) \rangle \]

\[\vec{F} = \langle 0, -125 \rangle \]

\[W = \langle 10\cos(20), 10\sin(20) \rangle \cdot \langle 0, -125 \rangle \]

\[W = (0)(10\cos(20)) + (-125)(10\sin(20))\]

\[W = -427.5 \text{ ft-lbs.} \]
8. Dr. Smith is hanging a sign for his medical practice. The sign is held by two support bars as shown in the figure. If the bars make a 60 degree angle with each other and the sign weighs 100 lbs., what are the magnitudes of the forces exerted by the sign on each support bar?

\[ F_1 \]
\[ \tan(60^\circ) = \frac{100}{||F_1||} \]
\[ ||F_1|| = \frac{100}{\tan(60^\circ)} \]
\[ ||F_1|| = 57.74 \]

\[ F_2 \]
\[ \sin(60^\circ) = \frac{100}{||F_2||} \]
\[ ||F_2|| = \frac{100}{\sin(60^\circ)} \]
\[ ||F_2|| = 115.47 \]
9. A plane flies due west at 250 km/hr while the wind blows south at 70 km/hr. Find the plane’s resultant velocity and direction.

\[ \theta = \tan^{-1} \left( \frac{70}{250} \right) \]

\[ \theta \approx 15.6^\circ \text{ south of due west} \]

\[ \frac{v}{r} = \sqrt{250^2 + 70^2} = 259.6 \text{ km/hr} \]
10. A plane flies east for 200 km, then 60 degrees south of east for 80 km. Find the plane's distance and direction from its starting point.

**Law of Cosines**

\[ a^2 = b^2 + c^2 - 2bc \cos(A) \]

\[ a = \sqrt{200^2 + 80^2 - 2(200)(80)\cos(120)} \]

\[ a = 249.8 \text{ km} \]

**Direction**

16.1° South of East

**Law of Sines**

\[ \frac{\sin(\theta)}{r} = \frac{\sin(120)}{249.8} \]

\[ \theta = \sin^{-1}\left(\frac{80\sin(120)}{249.8}\right) \]

\[ \theta = 16.1° \text{ South of Due East} \]
11. One force of 100 units acts on an object. Another force of 80 units acts on the object at a 40 degree angle from the first force. Find the magnitude and direction of the resultant force on the object.

\[ \vec{r} = \sqrt{100^2 + 80^2 - 2(100)(80)\cos 140} \]

\[ |\vec{r}| = 169.3 \text{ units} \]

\[ \frac{\sin \theta}{80} = \frac{\sin 140}{r} \]

\[ \theta \approx 17.7^\circ \]