For the exam, I'll have you do two decimals!

Honors Pre-Calculus

Law of Sines/Cosines, Right Triangles, and Area Review

Solve each triangle. Round your answers to the nearest tenth.

1) Law of Cosines

\[ A = 118.7^\circ \]
\[ B = 23.9^\circ \]
\[ C = 37.4^\circ \]

\[ 13^2 = 6^2 + 9^2 - 2(6)(9) \cos A \]

or

\[ 9^2 = 13^2 + 9^2 - 2(13)(9) \cos B \]

or

\[ 9^2 = 6^2 + 13^2 - 2(6)(13) \cos C \]

2) SAS Law of Cosines

\[ c^2 = 10^2 + 11^2 - 2(10)(11) \cos 108^\circ \]

\[ A = 38^\circ \]
\[ B = 34^\circ \]
\[ c = 17 \]

3) ASA Law of Sines

\[ \frac{32}{\sin 111^\circ} = \frac{b}{\sin 17^\circ} = \frac{a}{\sin 52^\circ} \]

\[ C = 111^\circ \]
\[ b = 10.02 \]
\[ a = 27.61 \]

4) AAS Law of Sines

\[ \frac{13}{\sin 34^\circ} = \frac{k}{\sin 109^\circ} = \frac{h}{\sin 37^\circ} \]

\[ H = 37^\circ \]
\[ K = 21.98 \]
\[ h = 13.99 \]

Solve each right triangle. Round answers to the nearest tenth.

5) SOHCAHTOA or Pythagorean Theorem

\[ B = 49 \]
\[ \tan(49^\circ) = \frac{X}{8} \]
\[ \sin(41^\circ) = \frac{8}{Y} \]

\[ X = 9.20 \]
\[ Y = 12.19 \]

6) 

\[ \cos(B) = \frac{2.3}{5} \]
\[ \sin(A) = \frac{2.3}{5} \]

\[ B = 62.61^\circ \]
\[ A = 77.39^\circ \]
Solve each triangle. Round your answers to the nearest tenth.

7) In \( \triangle CAB \), \( m \angle C = 39^\circ \), \( m \angle B = 14^\circ \), \( a = 33 \)

\[
\frac{33}{\sin(127)} = \frac{b}{\sin(14)} = \frac{c}{\sin(39)}
\]

\( A = 127^\circ \)
\( b = 10 \)
\( c = 26 \)

9) In \( \triangle EFD \), \( m \angle E = 130^\circ \), \( d = 31 \), \( e = 31 \)

\[
\frac{31}{\sin(130)} = \frac{31}{\sin D}
\]

\( D = 50 \)

\text{Not possible}
No Triangle Exists

Find the area of each triangle to the nearest tenth.

10) \( \triangle XYZ \)

\[
S = \frac{12+11+8}{2} = 15.5
\]

\( \text{Area} = 42.8 \)

12) \( \triangle ABC \)

\[
\text{Area} = \frac{1}{2}(9.7)(7)\sin 37
\]

\( \text{Area} = 20.4 \)

11) \( \triangle HKP \)

\[
\text{Area} = \frac{1}{2}(13)(7)\sin 112
\]

\( \text{Area} = 42.2 \)

13) \( \triangle STU \)

\[
S = \frac{8+6+4.8}{2} = 9.4
\]

\[
A = \frac{1}{2}(9.4)(9.4-4.8)(9.4-6)(9.4-8)
\]

\( \text{Area} = 14.3 \)
Law of Cosines/Sines/Right Triangle Review

1. Two ranger stations located 10km apart receive a distress call from a camper. Electronic equipment allows them to determine that the camper is at an angle of 71° from the first station and 100° from the second. Which station is closer to the camper? How far away is it?

\[ \frac{10}{\sin(71^\circ)} = \frac{x}{\sin(100^\circ)} = \frac{y}{\sin(71^\circ)} \]

- \( x = 6.295 \) km
- \( y = 60.44 \) km

Station 2 is closer.

60.44 km

2. Mr. Thompson observes a student trying to climb out of a second story window and “escape” from 3rd Period. The angle of elevation of his line of sight to the student is 22°. The angle of elevation of his line of sight to the top of the school is 46°. If Mr. Thompson stands 20 feet from the base of Cox Mill HS, how far from the top of the building is the student?

\[ \tan(22^\circ) = \frac{x}{20} \]

\[ x = 8.08 \text{ feet} \]

\[ \tan(46^\circ) = \frac{y}{20} \]

\[ y = 20.71 \text{ feet} \]

\[ z = 20.71 - 8.08 \]

\[ z = 12.63 \text{ feet} \]

3. The distances from a boat B to two seagulls on the shore are 100m and 80m respectively. If \( B=55^\circ \), how far would one seagull have to walk to meet the other seagull?

\[ X^2 = 100^2 + 80^2 - 2(100)(80)\cos(55^\circ) \]

\[ X^2 = 7223.78 \]

\[ X = 84.99 \text{ meters} \]

4. A damsel is in distress and is being held captive in a tower. Her knight in shining armor is on the ground below with a ladder. When the knight stands 15 feet from the base of the tower and looks up at his precious damsel, the angle of elevation to her window is 55 degrees. How long does the ladder need to be to reach the window?

\[ \cos(55^\circ) = \frac{15}{x} \]

\[ x \cdot \cos(55^\circ) = 15 \]

\[ x = 26.15 \text{ feet} \]
Law of Cosines/Sines/Right Triangle Review

5. Ships A and B leave port at the same time and sail on straight paths making an angle of 60° with each other. How far apart are the ships at the end of 3 hours if the speed of ship A is 25 km/h and that of ship B is 15 km/h?

\[ x^2 = 75^2 + 45^2 - 2(75)(45)\cos(60) \]
\[ x^2 = 4275 \]
\[ x = 65.38 \text{ km} \]

6. The sides of a triangle have lengths 10 cm, 9 cm, and 3 cm. Find the largest angle.

\[ 10^2 = 9^2 + 3^2 - 2(9)(3)\cos(A) \]
\[ 100 = 81 - 54\cos(A) \]
\[ 10 = -54\cos(A) \]
\[ -1.85185 = \cos(A) \]
\[ A = 100.67° \]

7. A 12 meter flagpole makes a 9 meter long shadow at a certain time of day. If you were to stand at the tip of the shadow, what would be the angle of elevation to the top of the pole?

\[ \tan(\theta) = \frac{12}{9} \]
\[ \theta = \tan^{-1}(12/9) \]
\[ \theta = 53.13° \]

8. Suppose you’re flying a kite and it gets caught at the top of a tree. You’ve let out all 100 feet of string from the reel and the angle that the string makes with the horizontal (the ground) is 15 degrees. If you are currently holding the kite string 4 feet above the ground, how tall is the tree?

\[ \sin(15) = \frac{x}{100} \]
\[ x = 25.88 \]
\[ + 4 \text{ feet} \]
\[ 29.88 \text{ feet} \]
Honors PreCalculus

Test Review - Vectors

Write each vector in component form.

1) $\overrightarrow{RS}$ where $R = (-2, 5)$ $S = (5, 3)$

2) $\overrightarrow{PQ}$ where $P = (6, 1)$ $Q = (0, -4)$

\[
\begin{align*}
\langle 5-(-2), 3-5 \rangle &= \langle 7, -2 \rangle \\
\langle 0-6, -4-1 \rangle &= \langle -6, -5 \rangle 
\end{align*}
\]

Find the magnitude for each vector.

3) $\overrightarrow{AB}$ where $A = (-10, 6)$ $B = (-7, -3)$

\[
\begin{align*}
\langle -7-(-10), -3-6 \rangle &= \langle 3, -9 \rangle \\
\text{Magnitude} &= \frac{3\sqrt{110}}{7} \text{ or } 9.49
\end{align*}
\]

Find the magnitude, angle, and compass direction for each vector.

4) $b = (12, -16)$

\[
\begin{align*}
\text{Magnitude} &= 20 \\
\tan \theta &= \frac{12}{16} \\
\theta &= 32.12^\circ \text{ South of East}
\end{align*}
\]

5) $b = (7, 37)$

\[
\begin{align*}
\tan \theta &= \frac{37}{7} \\
\theta &= 79.29^\circ \text{ North of East}
\end{align*}
\]

\[
\text{Magnitude} \approx 37.66
\]

6) $p = (-23, 44)$

\[
\begin{align*}
\tan \theta &= \frac{44}{-23} \\
\theta &= 60.4^\circ \text{ North of West}
\end{align*}
\]

\[
\text{Magnitude} \approx 49.65
\]
Write each vector in component form given the magnitude and angle.

7) \( |\mathbf{n}| = 80, 14^\circ \) South of West
   \[
   \begin{align*}
   \cos(14) &= \frac{x}{80} = 77.62 \\
   \sin(14) &= \frac{y}{80} = 19.35
   \end{align*}
   \]
   \( \langle -77.62, -19.35 \rangle \)

8) \( |\mathbf{v}| = 76, 10^\circ \) South of East
   \[
   \begin{align*}
   \cos(10) &= \frac{x}{76} \Rightarrow x = 74.84 \\
   \sin(10) &= \frac{y}{76} \Rightarrow y = 13.19
   \end{align*}
   \]
   \( \langle 74.84, -13.19 \rangle \)

Find the following information for each vector: Component form, magnitude and direction angle.

9) \( \overrightarrow{RS} \) where \( R = (-1, -2) \) \( S = (7, 10) \)
   \( \langle 8, 12 \rangle \)
   \[
   \begin{align*}
   \text{magnitude} &= 4\sqrt{13} \text{ or } 14.42 \\
   \tan \theta &= \frac{12}{8} \\
   \theta &= 56.31^\circ \text{ North of East}
   \end{align*}
   \]

10) \( \overrightarrow{AB} \) where \( A = (8, 5) \) \( B = (-1, -10) \)
    \( \langle -9, -15 \rangle \)
    \[
    \begin{align*}
    \text{magnitude} &= 3\sqrt{34} \text{ or } 17.49 \\
    \tan \theta &= \frac{15}{9} \\
    \theta &= 59.04^\circ \text{ South of West}
    \end{align*}
    \]

Find the component form of the resultant vector.

11) \( \mathbf{u} = \langle 9, -1 \rangle \)
    \( \mathbf{g} = \langle 11, -5 \rangle \)
    Find: \( \mathbf{u} + \mathbf{g} \)
    \( \langle 20, -6 \rangle \)

12) \( \mathbf{u} = \langle -6, 8 \rangle \)
    Find: \( 2\mathbf{u} \)
    \( \langle -12, 16 \rangle \)

Find the dot product of the given vectors.

13) \( \mathbf{u} = \langle -4, 2 \rangle \)
    \( \mathbf{v} = \langle -7, 9 \rangle \)
    \( (-4 \cdot -7) + (2 \cdot 9) \)
    \( 28 + 18 \]
    \( 46 \)

14) \( \mathbf{u} = \langle 2, -4 \rangle \)
    \( \mathbf{v} = \langle 3, 1 \rangle \)
    \( (2 \cdot 3) + (-4 \cdot 1) \)
    \( 6 - 4 \]
    \( 2 \)
1) After walking 11 km due north from camp, a hiker then turns 34° east and then walks 14 km to set up camp.

- **What is the resultant distance from the campsite to the hiker's original starting point?**

  \[ x^2 = (11)^2 + (14)^2 - 2(11)(14) \cos(146) \]

  \[ x = 23.92 \text{ Km} \]

2) Two boys push on a box. One pushes with a force of 125 Newtons to the east. The other exerts a force of 165 Newtons to the north. What is the size and direction of the resultant force on the box?

  \[ x^2 = 165^2 + 125^2 \]

  \[ \tan \theta = \frac{165}{125} \]

  \[ \text{Resultant Force} = 207 \text{ Newtons} \]

  \[ \theta = 52.85^\circ \text{ North of East} \]

3) Nadia is pulling a tarp along level ground with a force of 25 pounds directed along the tarp. If the tarp makes an angle of 50° with the ground, find the horizontal and vertical components of the force.

  \[ \cos(50) = \frac{x}{25} = 16.06 \text{ (Horizontal)} \]

  \[ \sin(50) = \frac{y}{25} = 19.15 \text{ (Vertical)} \]

4) Cara pulls the handle of a wagon at an angle of 46° along level ground with a force of 32 pounds. Find the position vector that represents her pulling the wagon.

  \[ \cos(46) = \frac{x}{32} \]

  \[ \sin(46) = \frac{y}{32} \]

  \[ \left< \begin{array}{c} 22.23 \ 
 23.02 \end{array} \right> \]

5) The speed of a powerboat in still water is 35 mi/h. It is traveling on a river that flows directly west at 8 mi/h. If the boat was trying to head directly north,

- **a)** What is the resulting speed of the boat (in mph)?

- **b)** What is the resulting direction of the boat? (Include angle and compass direction)

  \[ x^2 = 8^2 + 35^2 \]

  \[ \tan \theta = \frac{8}{35} \]

  \[ x = 35.9 \text{ mph} \]

  \[ \theta = 12.87^\circ \text{ West of North} \]

6) An airplane flies 350 miles due East and then turns 33° South to continue for 425 miles. What was the resultant distance?

  \[ x^2 = 350^2 + 425^2 - 2(350)(425) \cos(147) \]

  \[ x = 743.39 \text{ miles} \]