Exercise Set 1.1: An Introduction to Functions

Find the domain of each of the following functions. Then express your answer in interval notation.

1. \( f(x) = \frac{5}{x-3} \) \((-\infty, 3) \cup (3, \infty)\)

2. \( f(x) = \frac{x-6}{x+1} \) \((-\infty, -1) \cup (-1, \infty)\)

3. \( g(x) = \frac{x-4}{x^2-9} \) \((-\infty, -3) \cup (-3, 3) \cup (3, \infty)\)

4. \( f(x) = \frac{3x+1}{x^2+4} \) \((-\infty, \infty)\)

5. \( f(x) = \frac{x^2+6x+5}{x^2-11x+28} \) \((-\infty, 4) \cup (4, 7) \cup (7, \infty)\)

6. \( g(x) = \frac{3x^2+15}{x^2+8x-20} \) \((-\infty, -10) \cup (-10, 2) \cup (2, \infty)\)

7. \( f(x) = \frac{2}{\sqrt{x-4}} \) \((4, \infty)\)

8. \( f(x) = \sqrt{x-5} \) \([5, \infty)\)

9. \( g(x) = \sqrt{x+7} \) \([-7, \infty)\)

10. \( G(x) = \frac{\sqrt{x-2}}{x+4} \) \(x \leq \frac{3}{2} \) and \(x \neq -4\)

11. \( G(x) = \frac{\sqrt{x-3}}{x-7} \) \(x \geq 3\) and \(x \neq 7\)

12. \( g(x) = \frac{-3}{x^2-4} \) \((-\infty, -2) \cup (-2, 2) \cup (2, \infty)\)

13. \( f(x) = |x^2-4| \) \((-\infty, \infty)\)

Evaluate the following.

14. If \( f(x) = 5x-4 \), find:
   \( f(3), f\left(-\frac{1}{2}\right), f(a), f(a+3), f(a+3), f(a)+f(3) \)
   \[ f(3) = 13, \quad f\left(-\frac{1}{2}\right) = -\frac{13}{2}, \quad 5a-4, \quad 5a+11, \quad 5a-1, \quad 5a+7 \]

15. If \( f(x) = \frac{2+x}{x-3} \), find:
   \( f(-7), f(0), f\left(\frac{1}{2}\right), f(0), f\left(t^2-3\right) \)
   \[ f(-7) = -\frac{2}{3}, \quad f(0) = 0, \quad f\left(\frac{1}{2}\right) = \frac{-13}{12}, \quad \frac{2+t}{t-3}, \quad \frac{t^2-1}{t^2-6} \]

16. If \( g(x) = x^2-3x+4 \), find:
   \( g(0), g\left(-\frac{1}{2}\right), g(x+5), g\left(\frac{1}{a}\right), g(3a), 3g(a) \)
   \[ g(0) = 4, \quad g\left(-\frac{1}{2}\right) = \frac{7}{4}, \quad x^2+7x+14, \quad \frac{4a^2-3a+1}{a^2} \]
   \[ 9a^2-9a+4, \quad 3a^2-9a+12 \]
Transformations of Functions – Extra Practice

Write an equation for a function that has a graph with the given characteristics.

1. The shape of $y = x^2$ but is shifted left 3 units
   $$y = (x+3)^2$$

2. The shape of $y = \sqrt{x}$ but upside down and shifted right 3 units and up 4 units.
   $$y = -\sqrt{x-3} + 4$$

3. The shape of $y = |x|$ but stretched vertically by a factor of 2 and shifted right 3 units.
   $$y = 2|x-3|$$

4. The shape of $y = x^2$ but upside-down and shifted right 8 units
   $$y = -(x-8)^2$$

5. The shape of $y = |x|$ but stretched horizontally by a factor of 2 and shifted down 5 units
   $$y = \frac{1}{2} x - 5$$

6. The shape of $y = x^3$ but reflected across the x-axis and shifted up 1 unit.
   $$y = -x^3 + 1$$

7. The shape of $y = \sqrt{x}$ but reflected across the y axis, shifted down 2, and left 4.
   $$y = \sqrt{-x+4} - 2$$

8. The shape of $y = x^2$ but compressed vertically by a factor of 2 and shifted up 6.
   $$y = \frac{1}{2} x^2 + 6$$