Transformations of Functions

**Parent Functions**

You should be very familiar with these!

1. $f(x) = x^2$
   - Quadratic

2. $f(x) = x^3$
   - Cubic

3. $f(x) = \sqrt{x}$
   - Square Root

4. $f(x) = \frac{1}{x}$
   - Rational

5. $f(x) = |x|$
   - Absolute Value
Describe the transformation of the following functions and identify the parent graph. Sketch a graph for each.

1. $\sqrt{-x}$
   Reflect over the yaxis

2. $(x + 2)^2 - 4$
   Left 2
   Down 4

3. $(x - 1)^3 + 4$
   Right 1
   Up 4

4. $-|x|$
   Reflect over the xaxis

5. $-\sqrt{x+2}$
   Reflect over the xaxis
   Left 2

6. $\frac{1}{x+2}$
   Left 2
   Down 3
   (this moves the asymptotes)

7. $-(x)^3 + 4$
   Reflect over the yaxis
   Up 4

8. $-|x-6|$
   Reflect over the xaxis
   Right 6

(Just the transformed graph)
Describe the transformation. Sketch the parent function as well as the newly transformed graph.

\[ f(x) = \left( \frac{1}{4} x \right)^2 \]
Horiztonal Stretch by 4
(parabola gets wider)

\[ f(x) = (2x)^2 \]
Horizontal compression by 2
(parabola gets skinnier)

\[ f(x) = \frac{1}{4} x^2 \]
Vertical compression by 4
(parabola gets wider)

\[ f(x) = 3x^2 \]
Vertical stretch by 3
(parabola gets skinnier)
Write the following transformations using the parent function $y = |x|$

1) Reflect about the y axis and shift up 4 \[ y = | -x| + 4 \]

2) Reflect about the x axis and shift right 3 \[ y = -|x-3| \]

3) Vertically compressed by a factor of 3 and shift down 6 \[ y = \frac{1}{3} |x| - 6 \]

4) Horizontally compressed by a factor of 5 and shift up 2 \[ y = |5x| + 2 \]

Write the following using the parent function $y = x^2$

* Notice the square \((x)^2\) outside the parentheses when you have them!*

5) Horizontally compressed by a factor of 2 and shift up 9 \[ y = (2x)^2 + 9 \]

6) Vertically stretched by a factor of 2 and reflected over the x axis \[ y = -2x^2 \]

7) Vertically compressed by a factor of 4 and reflected about the y axis and shifted down 3 units \[ y = \frac{1}{4} (-x)^2 - 3 \]

Write an equation that represents the following transformations of $y = \sqrt{x}$

8) Horizontally stretched by a factor of 3 and up 6 \[ y = \sqrt{\frac{1}{3} x} + 6 \]

9) Vertically compressed by a factor of 4 and left 7 \[ y = \frac{1}{4} \sqrt{x + 7} \]

10) Vertically stretched by a factor of 2 and reflected about the x axis \[ y = -2\sqrt{x} \]

11) Horizontally compressed by a factor of 3, reflected about the y axis and shifted down 2 units \[ y = \sqrt{-3x} - 2 \]
Describe the transformation:

1. \( f(x) - 7 \)  \text{ down 7} \\
2. \( f(x + 4) \)  \text{ left 4} \\
3. \( f(x - 3) \)  \text{ right 3} \\
4. \( f(-x) - 1 \)  \text{ reflect y-axis, down 1} \\
5. \( f(x) + 5 \)  \text{ up 5} \\
6. \( -f(x) - 2 \)  \text{ reflect x-axis, down 2} \\
7. \( 2f(x) \)  \text{ V. stretch by 2} \\
8. \( \frac{1}{2} f(x) \)  \text{ V. Compress by 2} \\
9. \( f(3x) \)  \text{ H. Compress by 3} \\
10. \( f\left(\frac{x}{4}\right) \)  \text{ H. Stretch by 4} \\
11. \( 2f(x) - 3 \)  \text{ V. Stretch by 2, down 3} \\
12. \( \frac{1}{2} f(x - 4) \)  \text{ V. Compress. by 2, right 4} \\
13. \( -3f(x) \)  \text{ V. Stretch by 3, reflect x-axis}
Write the following using the parent function $y = f(x)$

1) Horizontally compressed by a factor of 2: $f(2x)$

2) Vertically stretched by a factor of 2: $2f(x)$

3) Horizontally stretched by a factor of 3: $f\left(\frac{1}{3}x\right)$

4) Vertically compressed by a factor of 4: $\frac{1}{4}f(x)$

5) Vertically stretched by a factor of 2 and reflected about the x-axis: $-2f(x)$

6) Horizontally compressed by a factor of 3 and reflected about the y-axis: $f(-3x)$

7) Vertically compressed by a factor of 4 and reflected about the y-axis: $\frac{1}{4}f(-x)$