1. \( y = -2 \sin \left( \frac{\theta}{2} + \frac{\pi}{2} \right) - 3 \)

- Amplitude: \(2\)
- Vertical Shift: \(-3\)
- Period: \(4\pi\)
- Phase Shift: \(-\pi\)

2. \( y = \frac{1}{2} \cos \left( 4x - 2\pi \right) + 1 \)

- Amplitude: \(\frac{1}{2}\)
- Vertical Shift: \(0\)
- Period: \(\frac{\pi}{2}\)
- Phase Shift: \(\frac{\pi}{2}\)

3. \( y = \cos \left( x + \frac{\pi}{4} \right) \)

- Amplitude: \(1\)
- Vertical Shift: \(-\frac{1}{2}\)
- Period: \(2\pi\)
- Phase Shift: \(-\frac{\pi}{4}\)
4.) \( y = 2 \sin (\theta) + 4 \)
   Amplitude: \( 2 \)  Vertical Shift: \( 4 \)  Period: \( \pi \)  Phase Shift: \( \_ \)
   - Start at 0
   - Increments of \( \frac{\pi}{2} \)

\[
\begin{array}{cccc}
0 & \frac{\pi}{2} & \pi & \frac{3\pi}{2} & 2\pi \\
4 & 6 & 4 & 2 & 4 \\
\end{array}
\]

5.) \( y = -\frac{1}{4} \sin (3\theta) \)
   Amplitude: \( \frac{1}{4} \)  Vertical Shift: \( \_ \)  Period: \( \frac{2\pi}{3} \)  Phase Shift: \( \_ \)
   - Start at 0
   - Increments of \( \frac{\pi}{6} \)

\[
\begin{array}{cccc}
0 & \frac{\pi}{6} & \frac{\pi}{3} & \frac{\pi}{2} & \frac{2\pi}{3} \\
0 & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\
\end{array}
\]

6.) \( y = \cos \left( x - \frac{\pi}{2} \right) \)
   Amplitude: \( 1 \)  Vertical Shift: \( \_ \)  Period: \( 2\pi \)  Phase Shift: \( \frac{\pi}{2} \)
   - Start at \( \frac{\pi}{2} \)
   - Increments of \( \frac{\pi}{2} \)

\[
\begin{array}{cccc}
\frac{\pi}{2} & \pi & \frac{3\pi}{2} & 2\pi & \frac{5\pi}{2} \\
1 & 0 & -1 & 0 & 1 \\
\end{array}
\]
Fill in the Blanks.

<table>
<thead>
<tr>
<th>Period</th>
<th>Midline</th>
<th>Positive</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Fourth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>Periodic</td>
<td>Negative</td>
<td>second</td>
<td>fourth</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. The __ of a periodic function is the horizontal line midway between the function’s maximum and minimum values.

2. A function that repeats itself at a regular interval or pattern is called __.

3. __ is the vertical distance between the midline and the maximum (or minimum) value of a function.

4. For the function $y = \sin x$, $2\pi$ is called the __.

5. Amplitude is always __.

6. The cosine function is positive in the __ and __ quadrants.

<table>
<thead>
<tr>
<th>Function</th>
<th>Amplitude</th>
<th>Period</th>
<th>Phase Shift</th>
<th>Midline</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. $y = 2 \cos x$</td>
<td>2</td>
<td>$2\pi$</td>
<td>—</td>
<td>$y = 0$</td>
</tr>
<tr>
<td>8. $y = \frac{1}{2} \sin 2x$</td>
<td>$\frac{1}{2}$</td>
<td>$\pi$</td>
<td>—</td>
<td>$y = 0$</td>
</tr>
<tr>
<td>9. $y = \cos (\frac{1}{2} x + 3\pi)$</td>
<td>1</td>
<td>$4\pi$</td>
<td>$-6\pi$</td>
<td>$y = 0$</td>
</tr>
<tr>
<td>10. $y = 4 \sin x$ - 3</td>
<td>4</td>
<td>$2\pi$</td>
<td>—</td>
<td>$y = -3$</td>
</tr>
<tr>
<td>11. $y = -3 \cos (\frac{30 - \pi}{2}) + 2$</td>
<td>3</td>
<td>$\frac{2\pi}{3}$</td>
<td>$\pi/6$</td>
<td>$y = 2$</td>
</tr>
<tr>
<td>12. $y = 4 \sin (\frac{30}{2} + 2\pi) - 7$</td>
<td>4</td>
<td>$4\pi$</td>
<td>$-4\pi$</td>
<td>$y = -7$</td>
</tr>
</tbody>
</table>

Write an equation for each of the following.

13. A cosine function that has amplitude 2 and a period $\pi$ and midline $y = -3$

14. A sine function that has amplitude $\frac{1}{2}$ and a period $2\pi$ and midline $y = 4$

15. A cosine function that has amplitude 3 and a period $\frac{\pi}{2}$ and phase shift of $\pi$.

16. A sine function that has amplitude $\frac{1}{2}$ and a period $\pi$, phase shift of $\frac{\pi}{4}$ and midline $y = 3$
$y = \sin \theta$
Period: $2\pi$
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$  

$y = \cos \theta$
Period: $2\pi$
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$ 

$y = \sec \theta = \frac{1}{\cos \theta}$ \hspace{1cm} \cos \theta \neq 0$
Period: $2\pi$
Domain: $x \neq \frac{n\pi}{2}$ where $n$ is an odd integer
Range: $(-\infty, -1] \cup [1, \infty)$

$y = \csc \theta = \frac{1}{\sin \theta}$ \hspace{1cm} \sin \theta \neq 0$
Period: $2\pi$
Domain: $x \neq n\pi$, where $n$ is an integer
Range: $(-\infty, -1] \cup [1, \infty)$
\[ y = \tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \text{cos} \theta \neq 0 \]

Period: \( \pi \)

Domain: \( x \neq \frac{n\pi}{2} \), where \( n \) is an odd integer

Range: \((-\infty, \infty)\)

\[ y = \cot \theta = \frac{\cos \theta}{\sin \theta}, \quad \sin \theta \neq 0 \]

Period: \( \pi \)

Domain: \( x \neq n\pi \), where \( n \) is an integer

Range:

1) Write the equation of a tangent function with a period of \( 2\pi \)
\[ \frac{\pi}{B} = 2\pi \quad B = \frac{1}{2} \quad y = \tan \left( \frac{1}{2} \theta \right) \]

2) Write the equation of a tangent function with a period of \( \frac{\pi}{3} \)
\[ \frac{\pi}{B} = \frac{\pi}{3} \quad B = 3 \quad y = \tan(3\theta) \]

3) Write the equation of a cosecant function with a period of \( \frac{2}{3} \pi \) and a phase shift of \( \frac{\pi}{3} \)
\[ \frac{2\pi}{B} = \frac{2\pi}{3} \quad -\frac{C}{3} = \frac{\pi}{3} \]
\[ B = 3 \quad -\frac{C}{3} = \frac{\pi}{3} \]
\[ C = -\pi \]
\[ y = \csc \left( 3\theta - \pi \right) \]
The table below shows the amount of electricity (in kilowatt hours, kWh) that Miss McCarthy used last year.

Plot each point on the graph shown:

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Value</td>
<td>732</td>
<td>870</td>
<td>728</td>
<td>414</td>
<td>458</td>
<td>585</td>
<td>656</td>
<td>835</td>
<td>684</td>
<td>419</td>
<td>461</td>
<td>667</td>
</tr>
</tbody>
</table>

What is the maximum kWh of electricity? **870**  Minimum kWh? **414**

Find the amplitude: **228**  midline: **y = 642**  period: **6**

Write a positive cosine function to model this scenario:

Since max is in February, phase shift of 2

\[ y = a \cos \left( \frac{\pi}{B} \theta - \frac{2\pi}{3} \right) + 642 \]

Using your equation, how many kilowatt hours of electricity would I anticipate using next September?

\[ \theta = 9 \]

\[ 756 \text{ kWh} \]

If each kWh of electricity costs $0.14, approximately how much will I spend on electricity in September?

\[ $105.84 \]

Why is my electricity bill so high in February? August?

February is really cold, so I turn up the heat.
August is really hot, so I turn up the air conditioner.
1) Abby is bouncing up and down on her pogo stick which is on a porch 4 feet off the ground. The highest she gets is 10 feet in the air, which she reaches in 2 seconds. She completes a bounce every 4 seconds.

Create a table that represents one full period:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7</td>
<td>10</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

Graph the function:

\[
\begin{align*}
\frac{2\pi}{B} &= 4 \quad B = \frac{\pi}{2} \\
y &= -3\cos\left(\frac{\pi}{2} \theta\right) + 7 \\
&\text{or}
\end{align*}
\]

\[
y = -3\cos(90 \theta) + 7
\]

a) In 45 seconds, what will be his height? Is he going up or down at this time?

\[
y = -3\cos(90(45)) + 7 = 7 \text{ feet}
\]

b) Within the first 45 seconds, how many times does he reach his peak?

\[
\frac{45 \text{ secs}}{4} = 11.25 \text{ cycles in 45 secs.} \quad \boxed{11 \text{ times}}
\]

2) The inside of David’s bicycle wheel has a diameter of 25 inches and is 3 inches off the ground. An ant has climbed inside the wheel. David starts riding the bicycle at a steady rate. In 0.8 seconds, the ant reaches its highest point on the wheel. The wheel makes a revolution every 1.6 seconds.

Determine 5 points and find an equation that describes the motion of the ant.

<table>
<thead>
<tr>
<th>0</th>
<th>.4</th>
<th>.8</th>
<th>1.2</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>15.5</td>
<td>28</td>
<td>15.5</td>
<td>3</td>
</tr>
</tbody>
</table>

Using your equation, determine the height of the ant after 25 seconds.

Amplitude = 12.5
Midline: \( y = 15.5 \)
Period: 1.6

\[
y = -12.5\cos\left(225 \theta\right) + 15.5 \quad \text{or} \quad y = -12.5\cos\left(\frac{5\pi}{4} \theta\right) + 15.5
\]

After 25 secs, 24.3 inches

If David rides for 60 seconds, how many revolutions has the ant made?

\[
\frac{60}{1.6} = \boxed{37.5 \text{ revolutions}}
\]