Happy National Cat Day!

- Park your phones
- Grab your calculators
- Take out HW from last night = Law of Sines & Right Triangle Trig

- Grab the Trig Applications ½ sheet.
Right Triangle Trigonometry - Applications

A 12 foot ladder makes a $25^\circ$ angle with the side of a building. How far away from the building is the base of the ladder?

From a point 100 feet from the base of a building, Angie looks up at a $40^\circ$ angle to the top of a building. She walks 20 feet closer to the building. At approximately what angle must Angie now look up to see the top of the building?

A man is standing on level ground 50 feet away from the wall of a building. He looks up at a window on the building. The angle of elevation to the bottom of the window is $28.5^\circ$. He then looks up at the top of the building. The angle of elevation to the top of the building is $35^\circ$. What is the approximate distance between the bottom of the window and the top of the building?
1. Law of Sines
\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]
or
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

2. Area formulas for \(\Delta's\) **MEMORIZE**
\[
A = \sqrt{s(s-a)(s-b)(s-c)}
\]
\[
A = \frac{1}{2}ab \sin C
\]
1. The safety instructions for a 20 ft ladder indicate that the ladder should not be included at more than a 70° angle with the ground. Suppose the ladder is leaned against a house at a 20°. Find the following:
   a. The distance from the base of the house to the foot of the ladder.
   
   b. How far up the side of the house the ladder can reach.

2. Find the measures of the acute angles of a 3 – 4 – 5 right triangle.

3. A student looks out of a second-story school window and sees the top of the school flagpole at an angle of elevation of 22°. The student is 18 ft above the ground and 50 ft from the flagpole. Find the height of the flagpole.

4) You are a block away from a skyscraper that is 780 feet tall. Your friend is between the skyscraper and yourself. The angle of elevation from your position to the top of the skyscraper is 42°. The angle of elevation from your friend’s position to the top of the skyscraper is 71°. To the nearest foot, how far are you from your friend?
Law of Sines and the Ambiguous Case

When given AAS or ASA, Law of Sines will always give one unique solution.

However, when given side-side-angle (SSA), then we could have one, two, or no solutions.

When dealing with the ambiguous case, remember:

1. In a triangle, the sum of the interior angles is 180°.
2. No triangle can have two obtuse angles.
3. The sine function has a range of \(-1 \leq \sin \theta \leq 1\).
4. If the \(\sin \theta = \) positive decimal < 1, the \(\theta\) can lie in the first quadrant (acute <) or in the second quadrant (obtuse <) with 2 answers.
**Example 1:** In triangle ABC, $a = 20$, $c = 16$, and $m\angle A = 30^\circ$.

\[
\sin 30^\circ = \frac{a}{b} = \frac{b}{c} = \frac{c}{a}
\]

\[
\sin 30^\circ = \frac{16}{b} \quad \text{and} \quad \sin C = \frac{c}{a} = \frac{16}{20} = 0.8
\]

\[
C = \sin^{-1}(0.8) = 53.13^\circ
\]

\[
180 - 30 - 53.13 = 96.87^\circ
\]

\[\frac{10}{\sin 30^\circ} = \frac{c}{\sin 96.87^\circ}\]

**Example 2:** In triangle ABC, $a = 7$, $c = 16$, and $m\angle A = 30^\circ$.

\[
\sin 30^\circ = \frac{a}{b} = \frac{b}{c} = \frac{c}{a}
\]

\[
\sin 30^\circ = \frac{7}{b} \quad \text{and} \quad \sin C = \frac{c}{a} = \frac{16}{7}
\]

\[b = \frac{7}{\sin 30^\circ} = 14 \quad \text{and} \quad \sin C = \frac{16}{7} \quad \text{cannot be greater than 1.}
\]

\[
\frac{180 - 30 - 96.87}{10} = 32.19
\]

**Example 3:** In triangle ABC, $a = 10$, $b = 16$, and $m\angle A = 30^\circ$.

\[
\sin 30^\circ = \frac{a}{b} = \frac{b}{c} = \frac{c}{a}
\]

\[
\sin 30^\circ = \frac{16}{b} \quad \text{and} \quad \sin C = \frac{c}{a} = \frac{16}{10} = 1.6
\]

\[C = \sin^{-1}(1.6) = 96.87^\circ
\]

\[180 - 30 - 96.87 = 53.13
\]

\[
\sin C = \frac{10}{\sin 30^\circ} = \frac{c}{\sin 96.87^\circ}
\]

\[\text{Triangle 1: } \quad B = 53.13^\circ, \quad C = 96.87^\circ, \quad \frac{10}{\sin 30^\circ} = \frac{c}{\sin 96.87^\circ}
\]

\[\text{Triangle 2: } \quad B = 126.87^\circ, \quad C = 23.13^\circ, \quad \frac{10}{\sin 30^\circ} = \frac{c}{\sin 23.13^\circ}
\]
Law of Sines and the Ambiguous Case Practice

Solve each triangle/triangles completely.

1. \( \angle A = 62^\circ, \ a = 30, \ b = 32 \)
2. \( \angle A = 16^\circ, \ a = 12, \ b = 37.5 \)

3. \( \angle C = 48^\circ, \ c = 91, \ a = 125 \)
4. \( \angle B = 112^\circ, \ b = 16.5, \ c = 5.4 \)

5. \( \angle C = 23.6^\circ, \ c = 9.8, \ a = 17 \)
6. \( \angle B = 155^\circ, \ b = 12.5, \ c = 8.4 \)

7. \( \angle A = 35^\circ, \ a = 11 \) and \( b = 15 \).