Exercise Set 1.1: An Introduction to Functions

Find the domain of each of the following functions. Then express your answer in interval notation.

1. \( f(x) = \frac{x}{x-3} \)
2. \( f(x) = \frac{x-6}{x+1} \)
3. \( g(x) = \frac{x-4}{x^2-9} \)
4. \( f(x) = \frac{3x+1}{x^2+4} \)
5. \( f(x) = \frac{x^2+6x+5}{x^2-11x+28} \)
6. \( g(x) = \frac{3x+15}{x^2-8x-20} \)
7. \( f(x) = \frac{2}{\sqrt{x-4}} \)
8. \( f(x) = \sqrt{x-5} \)
9. \( g(x) = \sqrt{x+7} \)
10. \( f(x) = \frac{\sqrt{x+5}}{x+4} \)
11. \( g(x) = \frac{\sqrt{x+2}}{x-3} \)
12. \( g(x) = \frac{-3}{\sqrt{x^2-4}} \)
13. \( f(x) = |x^2 - 4| \)
14. If \( f(x) = 5x - 4 \), find:
   \[ f(0), f\left(\frac{1}{2}\right), f(5), f(a + 3), f(a) + 3, f(a) + f(3) \]
15. If \( f(x) = \frac{2x + 3}{x - 5} \), find:
   \[ f(-3), f(0), f\left(\frac{1}{2}\right), f(0), f\left(\frac{3}{2}\right) \]

16. If \( g(x) = x^2 - 3x + 4 \), find:
   \[ g(0), g\left(\frac{1}{2}\right), g(-3), g\left(\frac{1}{3}\right), g(\pi), g(-\pi), g(3) \]
Transformations of Functions – Extra Practice

Write an equation for a function that has a graph with the given characteristics.

1. The shape of \( y = x^2 \) but is shifted left 3 units.

2. The shape of \( y = \sqrt{x} \) but upside down and shifted right 3 units and up 4 units.

3. The shape of \( y = |x| \) but stretched vertically by a factor of 2 and shifted right 3 units.

4. The shape of \( y = x^2 \) but upside-down and shifted right 8 units.

5. The shape of \( y = |x| \) but stretched horizontally by a factor of 2 and shifted down 5 units.

6. The shape of \( y = x^1 \) but reflected across the x-axis and shifted up 1 unit.

7. The shape of \( y = \sqrt{x} \) but reflected across the y-axis, shifted down 2, and left 4.

8. The shape of \( y = x^2 \) but compressed vertically by a factor of 2 and shifted up 6.
Exercise Set 1.1: An Introduction to Functions

Find the domain of each of the following functions. Then express your answer in interval notation.

1. \( f(x) = \frac{8}{x-3} \quad x \neq 3 \)
2. \( f(x) = \frac{x-6}{x+1} \quad x \neq -1 \)
3. \( g(x) = \frac{x-4}{x^2-9} \quad x \neq \pm 3 \)
4. \( f(x) = \frac{3x+1}{x^2+4} \quad \mathbb{R} \)
5. \( f(x) = \frac{x^2+6x-5}{x^2-11x+28} \quad x \neq 7 \text{ or } 4 \)
6. \( g(x) = \frac{3x+15}{x^2+6x-20} \quad x \neq 2 \text{ or } -10 \)
7. \( f(x) = \frac{2}{\sqrt{x-4}} \quad x > 4 \)
8. \( f(x) = \sqrt{x-5} \quad x \geq 5 \)
9. \( g(x) = \sqrt{x+7} \quad x \geq -7 \)
10. \( f(x) = \frac{\sqrt{2x+7}}{2x+4} \quad x \geq -\frac{3}{2} \text{ or } x < -4 \)
11. \( g(x) = \frac{\sqrt{2x-3}}{x-7} \quad x \geq 3 \text{ or } x < 7 \)
12. \( g(x) = \frac{-3}{\sqrt{x^2-4}} \quad x > 2 \text{ or } x < -2 \) (use the sign test)
13. \( f(x) = |x^2-4| \quad \mathbb{R} \)

Evaluate the following.

14. If \( f(x) = 3x-4 \), find:
   \( f(0), f(\frac{1}{2}), f(a), f(a+3), f(a)+3, f(a)+f(0) \)
12. \(-\frac{13}{2}, 5a-4, 5a+11, 5a-1, 5a+8 \)

15. If \( g(x) = x^2-3x+4 \), find:
   \( g(0), g(\frac{1}{2}), g(a-5), g(a+1), g(x^2), 3g(a) \)
6. \( 4, \frac{77}{16}, x^2+7x+14 \)
\( \frac{4a^2-3a+1}{a^2}, 9a^2-9a+4, 3a^2-9a+12 \)
Transformations of Functions — Extra Practice

Write an equation for a function that has a graph with the given characteristics.

1. The shape of \( y = x^2 \) but is shifted left 3 units \( y = (x + 3)^2 \)

2. The shape of \( y = \sqrt{x} \) but upside down and shifted right 3 units and up 4 units.
   \[ y = -\sqrt{x - 3} + 4 \]

3. The shape of \( y = \left| x \right| \) but stretched vertically by a factor of 2 and shifted right 3 units.
   \[ y = 2 \left| x - 3 \right| \]

4. The shape of \( y = x^3 \) but upside-down and shifted right 8 units.
   \[ y = -(x - 8)^3 \]

5. The shape of \( y = \left| x \right| \) but stretched horizontally by a factor of 2 and shifted down 5 units.
   \[ y = \left| \frac{x}{2} \right| - 5 \]

6. The shape of \( y = x^3 \) but reflected across the \( x \)-axis and shifted up 1 unit.
   \[ y = -x^3 + 1 \]

7. The shape of \( y = \sqrt{x} \) but reflected across the \( y \)-axis, shifted down 2, and left 4.
   \[ y = -\sqrt{x + 4} - 2 \]

8. The shape of \( y = x^2 \) but compressed vertically by a factor of 2 and shifted up 6.
   \[ y = \frac{1}{2} x^2 + 6 \]
Pre-Calculus Warmup:

Given that \( h(x) = \frac{x+4}{x} \)

a. Find \( h(9) = \frac{(9)+4}{9} = \frac{9+4}{9} = \frac{13}{9} \)

b. Find \( h \left( \frac{m+2}{5} \right) = \frac{\left( \frac{m+2}{5} \right) + 4 \left( \frac{5}{4} \right)}{m+2} = \frac{\frac{m+2}{5} + \frac{20}{5}}{\frac{m+2}{5}} = \frac{m+2}{5} + \frac{20}{5} \)

\[ \frac{m+22}{5} \]

Find the domain of the following function:

\[ y = \frac{7x+2}{\sqrt{x^2-6x-16}} \]

1. \( \sqrt{real} \)
2. Can't by "0"

\[ x^2 - 6x - 16 > 0 \]
\[ (x-8)(x+2) > 0 \]
\[ x = 8 \quad x = -2 \]

\[ \begin{array}{cccc}
-2 & 8 & \hline
+ & - & + \\
\end{array} \]

\[ (3)^2 - 6(-3) - 16 > 0 \]
\[ 11 > 0 \quad \text{True} \]

\[ (a)^2 - (a) - 16 > 0 \]
\[ (9)^2 - (9) - 16 > 0 \]
\[ 70 \quad \text{False} \]

\[ D : (-\infty, -2) \cup (8, \infty) \]
Pre-Calculus Warmup:

Given that $h(x) = \frac{x+4}{x}$

a. Find $h(9) = \frac{9+4}{9} = \frac{13}{9}$

b. $h\left(\frac{m+2}{5}\right) = \frac{\frac{m^2 + 2}{5} + 4}{\frac{m+2}{5}} = \frac{\frac{m^2 + 2 + 20}{5}}{\frac{m+2}{5}} = \frac{\frac{m^2 + 22}{5}}{\frac{m+2}{5}}$

$= \frac{m^2 + 22}{5} \cdot \frac{5}{m+2} = \frac{m^2 + 22}{m+2}$

Find the domain of the following function: $y = \frac{7x+2}{\sqrt{x^2 - 6x - 16}}$

Quadratic under radical

Use sign test:

$x^2 - 6x - 16 > 0$ (not = 0 b/c in den.)

$(x-8)(x+2) > 0$

$x = 8$ $x = -2$

Domain: $(-\infty, -2) \cup (8, \infty)$