8.4 Perpendicular Vectors

The inner product of \( \mathbf{a} = \langle x_1, y_1 \rangle \) and \( \mathbf{b} = \langle x_2, y_2 \rangle \) is \( \mathbf{a} \cdot \mathbf{b} = x_1x_2 + y_1y_2 \). The inner product is denoted by a dot between the two vectors. Note that the inner product is a scalar quantity.

Two vectors are perpendicular if and only if their inner product (also called the dot product) is zero. Perpendicular vectors are also called orthogonal vectors.

Find the inner (dot) products

1. \( \langle 5, -3 \rangle \cdot \langle 4, 6 \rangle = 20 + (-18) = 2 \) 
   \[ \text{No} \quad \perp \]

2. \( \langle 4, 6 \rangle \cdot \langle -3, 2 \rangle = -12 + 12 = 0 \) 
   \[ \text{No} \quad \perp \]

1. If \( \mathbf{a} = \langle 2, -5 \rangle \), \( \mathbf{b} = \langle 4, 1 \rangle \), \( \mathbf{c} = \langle 10, 4 \rangle \), ... is \( \mathbf{a} \) perpendicular to \( \mathbf{b} \)?

\[ \mathbf{a} \cdot \mathbf{b} = (2)(4) + (-5)(1) = 3 \quad \text{No} \quad \perp \]

... is \( \mathbf{a} \) perpendicular to \( \mathbf{c} \)?

\[ \mathbf{a} \cdot \mathbf{c} = (2)(10) + (-5)(4) = 0 \quad \text{Yes} \quad \perp \]

2. If \( \mathbf{a} = \langle -4, 1, 0 \rangle \) and \( \mathbf{b} = \langle 5, 4, -2 \rangle \), is \( \mathbf{a} \) perpendicular to \( \mathbf{b} \)?

\[ \mathbf{a} \cdot \mathbf{b} = (-4)(5) + (1)(4) + (0)(-2) \]
\[ = -20 + 4 + 0 = -16 \quad \text{Yes} \quad \perp \]

Find the determinant of the matrix.

\[
\begin{vmatrix}
5 & 6 \\
2 & 3
\end{vmatrix}
\]

\[ \det = 15 - 12 = 3 \]

Cross Products: the cross product of two vectors is a vector; this vector does not lie in the plane of the 2 vectors, but it is perpendicular to that plane; the cross-product is denoted by an "x" between the 2 vectors.

If \( \mathbf{a} \times \mathbf{b} = \mathbf{c} \), then \( \mathbf{a} \perp \mathbf{c} \) and \( \mathbf{b} \perp \mathbf{c} \).

(the formula for the cross product is on p. 507)
\( \vec{a} = \langle 0, 3, 1 \rangle \) and \( \vec{b} = \langle 0, 1, 2 \rangle \)

1. Find \( \vec{a} \times \vec{b} \) and verify that the resulting vector is perpendicular to the given vectors.

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 3 & 1 \\
0 & 1 & 2 \\
\end{vmatrix} = \mathbf{a} \times \mathbf{b}
\]

\[
\begin{vmatrix}
1 & 3 & 1 \\
0 & 3 & 2 \\
0 & 1 & 2 \\
\end{vmatrix} = \det = 5
\]

\[
\begin{vmatrix}
1 & 0 & 1 \\
0 & 3 & 2 \\
0 & 1 & 2 \\
\end{vmatrix} = \det = 0
\]

\[
\begin{vmatrix}
1 & 0 & 1 \\
0 & 3 & 2 \\
0 & 1 & 2 \\
\end{vmatrix} = \det = 0
\]

\[
5\mathbf{i} - 0\mathbf{j} + 0\mathbf{k} \leq 5, 0, 0
\]

2. Find \( \vec{b} \times \vec{a} \)

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 1 & 2 \\
0 & 3 & 1 \\
\end{vmatrix} = \mathbf{b} \times \mathbf{a}
\]

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-5 & 0 & 0 \\
1 & 6 & 0 \\
\end{vmatrix} = \det = 5
\]

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-5 & 0 & 0 \\
-5 & 0 & 0 \\
\end{vmatrix} = \det = 0
\]

\[
\mathbf{b} \times \mathbf{a} = \langle -5, 0, 0 \rangle
\]