Partial Fraction Decomposition

\[
\frac{8}{x+1} - \frac{5}{x-4} \left( \frac{x+1}{x+1} \right)
\]

\[
\frac{8x-32}{(x-4)(x+1)} - \frac{5x+5}{(x-4)(x+1)}
\]

\[
\frac{8x-32-5x-5}{x^2-3x-4}
\]

\[
\frac{3x-37}{x^2-3x-4}
\]

The fractions \(\frac{3}{x+1}\) and \(\frac{5}{x-4}\) are called partial fractions.

We want to take the rational function \(\frac{P(x)}{Q(x)}\) and break it into its partial fractions. This process is called partial fraction decomposition.

Goal:

\[
\frac{3x-37}{x^2-3x-4} = \frac{3}{x+1} - \frac{5}{x-4}
\]
1. **Divide if improper**: If \( N(x)/D(x) \) is an improper fraction, divide the denominator into the numerator to obtain

\[
\frac{N(x)}{D(x)} = \text{polynomial} + \frac{N_1(x)}{D(x)}
\]

\( \frac{N_1(x)}{D(x)} \) is the function we need to decompose

2. **Factor the denominator**: Completely factor the denominator into linear factors and/or quadratic factors

3. Based on the number and types of factors, we can set up a sum of fractions. See the table below.

<table>
<thead>
<tr>
<th>Factor in denominator</th>
<th>Term in partial fraction decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ax + b )</td>
<td>( \frac{A}{ax + b} ) \text{ Linear}</td>
</tr>
<tr>
<td>( (ax + b)^k )</td>
<td>( \frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \cdots + \frac{A_k}{(ax + b)^k} ) \text{ Repeated Linear}</td>
</tr>
<tr>
<td>( ax^2 + bx + c )</td>
<td>( \frac{Ax + B}{ax^2 + bx + c} ) \text{ Linear}</td>
</tr>
<tr>
<td>( (ax^2 + bx + c)^k )</td>
<td>( \frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k} ) \text{ Repeated Quadratic}</td>
</tr>
</tbody>
</table>
1. Factor Denominator

Distinct Linear Factors

Set-up the partial fractions, but do not solve.

\[
\frac{x-2}{x^2-x-12} = \frac{A}{x-4} + \frac{B}{x+3}
\]

\[
\text{Distinct Linear}
\]

\[
\frac{5x-8}{x^3+3x^2+2x} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x+1}
\]

\[
\text{Distinct Linear}
\]

\[
\frac{x^2+6}{x^2+2x-8} = \frac{A}{x+4} + \frac{B}{x-2}
\]

\[
\text{Distinct}
\]

\[
\frac{5x-2}{x^3-5x^2+6x} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x-2}
\]

\[
\text{Distinct}
\]

\[
** \frac{2x+3}{x^4-36} = \frac{A}{x^2+6} + \frac{B}{x^2-6}
\]

\[
\text{Distinct Quadratic}
\]
Repeated Linear Factors – Setup the partial fractions but do not solve.

\[
\frac{3x-41}{(x^2+6x+9)(x-2)} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-2}
\]

\[
\frac{7x-20}{(x-4)(x-4)(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{(x-4)^3} + \frac{D}{x+2}
\]

\[
\frac{3x^2+5}{(x^2-8x+16)(x+1)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x+1}
\]

Distinct Quadratic Factors – Setup the partial fractions but do not solve

\[
\frac{x-42}{x^3+7x^2+12} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+4}
\]

\[
\frac{4x^2+1}{x^3+5x} = \frac{A}{x} + \frac{Bx+C}{x^2+5}
\]
Partial Fraction Decomposition

Find the partial fraction decomposition for each...do not solve for the constants, just set them up!

1) \( \frac{3x-1}{x^2-x} \)
   Answer: 

2) \( \frac{-6x+3}{x^2-6} \)
   Answer: 

3) \( \frac{-7x^2-4x-14}{(x^2+1)(x-2)} \)
   Answer: 

4) \( \frac{-3x-29}{(x-7)(x+3)} \)
   Answer: 

5) \( \frac{12x-7}{(2x+1)(x-2)} \)
   Answer: 

6) \( \frac{7x^2-16x+36}{x^2-16} \)
   Answer: 

7) \( \frac{2x^3-3x^2}{(x+3)(x^2+4x+6)} \)
   Answer: 

8) \( \frac{x}{16x^4-1} \)
   Answer: 

9) \( \frac{-x^2+12x-5}{(x+1)(x^2+5)} \)
   Answer: 

10) \( \frac{x^3}{(x+2)^2(x-2)^2} \)
    Answer: 

Name: ________________________________