Linear Functions – Mathematical Modeling

Graph $f(x) = 3x - 6$
- Slope $m = 3$
- $y$-intercept $y = -6$

Graph $f(x) = 5 - 2x$
- Slope $m = -2$
- $y$-intercept $y = 5$

Example: A salesman earns $W(n) = 300 + 75n$, where $n$ is the number of products sold, and $W(n)$ is the total weekly wage in dollars.

a) Identify the slope and $y$-intercept and explain their meaning in practical terms.

- The salesman earns $300 even if he sells zero products. So $300 is his base salary ($y$-intercept).
- He then earns $75 extra for every product he sells (slope).

Example: Sam purchased a car that was worth $15,800 in 2008. Three years later, the car is worth $6800. Assuming that the car depreciates linearly, write the equation that models this relationship (hint: use $x = 0$ for 2008).

- Sam bought the car for an initial price of $15,800 ($y$-intercept).
- The car is losing value at a rate of $3000 per year (slope).

$y = 15800 - 3000x$
1. Your salary was $28,500 in 1994 and $32,900 in 1996. If your salary follows a linear growth pattern, what will your salary be in 2012?

\[ y = 2000x + 28500 \]

a. What is the slope of this model? What does it represent?

Your salary is increasing $2000 per year.

2. A small college had 2546 students in 2003 and 2702 in 2005. If the enrollment follows a linear growth pattern, how many students did the college have in 2009?

\[ y = 78x + 2546 \]

a. What is the slope of this model? What does it represent?

College enrollment is increasing 78 students per year.

3. A small business purchases a piece of equipment for $875. After 5 years the equipment will be outdated and have no value. Write a linear equation giving the value \( V \) of the equipment during the 5 years it will be used.

\[ V = 875 - 175x \]

a. What is the slope of this model? What does it represent?

Equipment loses its value $175 per year.

4. A small business purchases a piece of equipment for $25,000. After 10 years the equipment will have to be replaced. Its value at that time is estimated to be $2000. Write a linear equation giving the value \( V \) of the equipment during the 10 years it will be used.

\[ V = 25000 - 2300x \]

a. What is the slope of this model? What does it represent?

Equipment is losing its value $2300 per year.

5. A manufacturer pays its assembly line workers $11.50 per hour. In addition, workers receive a piecework rate of $0.75 per unit produced. Write a linear equation for the hourly wages \( W \) in terms of the number of units \( x \) produced per hour.

\[ W = 11.50 + 0.75x \]

6. A real estate office handles an apartment complex with 50 units. When the rent per unit is $580 per month, all 50 units are occupied. However, when the rent is $625 per month, the average number of occupied units drops to 47. Assume the relationship between the monthly rent \( p \) and the demand \( D \) is linear.

a) Write a linear equation giving the demand \( D \) in terms of the rent \( p \).

\[ D = -106.6x + 5866 \]

b) Use this equation to predict the number of units occupied if the rent is $655.

\[ D = 45 \]

c) If the demand drops to 42 units, what is the corresponding rent?

$700

7. Three consecutive integers have a sum of 87. Find the integers.

\[ 29, 39, 30 \]

8. A rectangle has a length that is 5 units longer than the width. The perimeter is 58 units. Find the dimensions.

\[ 2x + 2x + 10 = 58 \]

\[ x = 12 \]

\[ 12 \times 17 \]