Oblique $\triangle$

**Law of Sines**
- ASA
- AAS
- SSA *
  - Ambiguous Case
  - (Find 2 $\triangle$s)

**Law of Cosines**
- SAS
- SSS
**Law of Cosines:** In any triangle ABC, with side lengths a, b, c

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ b^2 = a^2 + c^2 - 2ac \cos B \]
\[ c^2 = a^2 + b^2 - 2ab \cos C \]

If we want to find missing sides or missing angles,

We will use the Law of Cosines when we are given \[ \text{SAS (side-angle-side) or SSS (side-side-side)} \]

*If you start with Law of Cosines, finish with Law of Cosines*
Example 1: Solve the triangle (find all sides and angles)

\[ b^2 = a^2 + c^2 - 2ac \cos B \]
\[ b = \sqrt{(16)^2 + (10)^2 - 2(16)(10)\cos(110)} \]
\[ b = 21.57 \]

\[ c^2 = a^2 + b^2 - 2ab \cos C \]
\[ 100 = (16)^2 + (21.57)^2 - 2(16)(21.57)\cos C \]
\[ -721.26 = -690.24 \cos C \]
\[ \cos^{-1}(-0.9) = C \]
\[ 25.84^\circ = \angle C \]

\[ A = 44.16^\circ \]
Example 2: Solve the triangle.

\[ 20^2 = 15^2 + 8.3^2 - 2(15)(8.3) \cos A \]
\[ 400 = 225 + 68.89 - 249 \cos A \]
\[ 106.11 = -249 \cos A \]
\[ -0.426 = \cos A \]
\[ \cos^{-1}(-0.426) = A \]
\[ 115.22^\circ = A \]

\[ 8.3^2 = 15^2 + 20^2 - 2(15)(20) \cos B \]
\[ 68.89 = 225 - 600 \cos B \]
\[ -556.11 = -600 \cos B \]
\[ -\frac{556.11}{-600} \cos B \]
\[ 0.927 = \cos B \]
\[ \cos^{-1}(0.927) = B \]
\[ 22.03^\circ = B \]

\[ \Delta \text{Sum} \]
\[ A = 42.75^\circ \]
\[ 180 - 22.03 - 115.22 = 42.75 \]
(Extra) Example 3: Use the Law of Cosines to solve the triangle.

(Extra) Example 4: Use the Law of Cosines to solve the triangle.
Law of Cosines Practice

Solve each triangle using Law of Cosines. Round side lengths and angles to the nearest tenth.

1. \( a = 14, \ c = 20, \ B = 38^\circ \)

2. \( A = 60^\circ, \ c = 17, \ b = 12 \)

3. \( a = 4, \ b = 6, \ c = 3 \)

4. \( A = 103^\circ, \ b = 31, \ c = 52 \)

5. \( a = 15, \ b = 26, \ C = 132^\circ \)

6. \( a = 31, \ b = 52, \ c = 43 \)