Happy Friday the 13th!
- Park your phones
- Grab your calculators - clear desk
15 minute max test.
15 minutes max

When done, put test in bin and grab warm up
Deltamath.com
Create student account
Teacher number is 419223
WARM UP

1. Simplify:
\[
\frac{4b^2(b-2)}{4b^3 - 8b^2} \cdot \frac{2}{2} \cdot \frac{b^2 - 4}{b^2 - 4} = \frac{4b^2}{a(b-a)(b+a)}
\]

2. Simplify:
\[
\frac{(b-4)(b-7)}{b^2 - 11b + 28} \cdot \frac{5(b-4)}{5b - 20} = \frac{(b-4)(b-7)(b+10)}{(b-7)(5)(b-4)}
\]
\[
\frac{b+10}{5} = \frac{1}{5}b + 2 = \frac{b}{5} + 2
\]
Adding and Subtracting Rational Expressions

1) \(\frac{-6}{x-2} + \frac{5}{(x-2)^2}\) 

2) \(\frac{-3}{x-6} + \frac{5}{x+4}\)

3) \(\frac{2}{x-3} + \frac{8}{x} + \frac{4}{x+1}\)

4) \(\frac{-3}{2x+3} - \frac{7}{3x+5}\)
Graphing Rational Functions:

A rational function, \( r(x) \), is a function of the form:

\[
 r(x) = \frac{P(x)}{Q(x)}
\]

where \( P(x) \) and \( Q(x) \) are polynomial functions.

Rational functions often have asymptotes:

- **Vertical Asymptote**: \( x = \text{constant} \)
  - Example: \( x = 2 \)

- **Horizontal Asymptote**: \( y = \text{constant} \)
  - Example: \( y = 2 \)

- **Slant Asymptote**: \( y = mx + b \)
  - Example: \( y = x - 1 \)

The most elementary of the rational functions is:

\[
 f(x) = \frac{1}{x}
\]

What is the horizontal asymptote? \( y = 0 \)

What is the vertical asymptote? \( x = 0 \)

What causes the vertical asymptote? *Can't divide zero* "True" zeros in denominator.
**Vertical Asymptotes and Holes in the Graph**

Let \( r(x) = \frac{P(x)}{Q(x)} \) be a rational function with polynomials \( P(x) \) and \( Q(x) \)

**Vertical Asymptotes:** The vertical asymptotes are the vertical line \( x = a \) where \( a \) is a real zero of the denominator only.

**Holes:** The holes of a function can be found when the numerator and denominator have a common factor. The real zero of that common factor would create a hole in the graph.

Identify the domain and then any holes or vertical asymptotes

1. \( f(x) = \frac{x+2}{x^2+5x+6} \)
   - Domain: \( x \neq -3, x \neq -2 \)
   - VA: \( x = -3 \)
   - Holes: \( (-2, 1) \)

2. \( f(x) = \frac{x+1}{2x^2+7x+5} \)
   - Domain: \( x \neq \frac{-5}{2}, x \neq -1 \)
   - VA: \( x = -\frac{5}{2} \)
   - Holes: \( (-1, \frac{1}{3}) \)

3. \( f(x) = \frac{3x^2}{x^2-16} \)
   - Domain: \( x \neq -4, x \neq 4 \)
   - VA: \( x = -4, x = 4 \)
   - Holes: none

***How is Domain related to VA’s and Holes?***

VA & holes are parts of the restricted domain.

Points of Discontinuity
Simplify each expression and identify any holes or vertical asymptotes.

1) \( \frac{n^2 + 10n + 25}{n + 5} \)  \[ \text{hole } n = -5 \] \((-5, 0)\)

2) \( \frac{10}{10b - 15} = \frac{10}{5(2b - 3)} \) \(2b - 3\)  \(b = \frac{3}{2}\)

3) \( \frac{15t - 40}{35r + 40} \)

4) \( \frac{2n^2 - 19n + 24}{5n^2 - 47n + 56} \)

5) \( \frac{7m^2 + 7m - 14}{5m^3 + 16n + 12} \)

6) \( \frac{4p^2 + 20p}{7p^3 + 34p - 5} \)

7) \( \frac{3x^2 + 2x - 21}{7x + 21} \)

8) \( \frac{9x^2 - 15x}{21x^2 - 18x} \)
**Horizontal Asymptotes**

1) When the degree of the denominator is GREATER than the numerator, then the horizontal asymptote is \( y = 0 \) (the x-axis)

   "Bottom bigger, yo!"

2) When the degree of the denominator is LESS than the numerator, then there is no horizontal asymptote

   "Top bigger, no... But, there maybe a Slant."

3) When the degree of the denominator and numerator are equal, then the horizontal asymptote is \( y = \frac{a}{b} \) where \( a \) and \( b \) are the lead coefficients.

Identify the horizontal asymptote, if any.

\[
\begin{align*}
f(x) &= \frac{2x+1}{3x-5} \quad & f(x) &= \frac{5x^2+1}{2x-4} \quad & f(x) &= \frac{2x^2+6}{8x^3-7} \\
match \quad y &= \frac{2}{3} \quad & no \ HA, \ but \ maybe \ a \ Slant. \quad & bottom \ bigger \quad y=0
\end{align*}
\]

\[
\begin{align*}
f(x) &= \frac{3x^2+1}{6x-4} \quad & f(x) &= \frac{4x^3+1}{3x^3-5} \quad & f(x) &= \frac{3x+1}{9x^4+2} \\
no, \ but \ maybe \ a \ Slant. \quad & y &= \frac{4}{3} \quad & y=0
\end{align*}
\]
Rational Functions usually end in branches

Graphing Guided Practice:

Identify any asymptotes and the sketch a graph of the function:

Graph $f(x) = \frac{5x+10}{x-4}$

Vertical Asymptotes: $x = 4$

Holes: none

Horizontal Asymptote: $y = 5$

x-intercepts: $(-2, 0)$

y-intercept: $(0, \frac{5(0)+10}{0-4}) = (-\frac{5}{2}, 0)$

Graph: $f(x) = \frac{x^2-9}{x-3}$

Vertical Asymptotes: none

Holes: $(3, 6)$

Horizontal Asymptote: none

x-intercepts: $(-3, 0)$

y-intercept: $(0, 3)$
Identify any vertical, horizontal, and/or slant asymptotes for each of the following functions. Then create a sketch of the graph.

1) \( f(x) = \frac{x+5}{x-2} \)  

VA: ___________________  hole(s): ___________________

HA: ___________________

x-int: ___________________  y-int: ___________________

2) \( f(x) = \frac{-4x+8}{2x+3} \)  

VA: ___________________  hole(s): ___________________

HA: ___________________

x-int: ___________________  y-int: ___________________

3) \( f(x) = \frac{3x+6}{2x-1} \)  

VA: ___________________  hole(s): ___________________

HA: ___________________

x-int: ___________________  y-int: ___________________

4) \( f(x) = \frac{(x-2)(x+3)}{(x-2)(x-4)} \)  

VA: ___________________  hole(s): ___________________

HA: ___________________

x-int: ___________________  y-int: ___________________

5) \( f(x) = \frac{(6-x)(x+3)}{(x-2)(x+3)} \)  

VA: ___________________  hole(s): ___________________

HA: ___________________

x-int: ___________________  y-int: ___________________

6) \( f(x) = \frac{x^2+x-20}{x-4} \)  

VA: ___________________  hole(s): ___________________

HA: ___________________

x-int: ___________________  y-int: ___________________

7) \( f(x) = \frac{x^2-3x-10}{x-5} \)  

VA: ___________________  hole(s): ___________________

HA: ___________________

x-int: ___________________  y-int: ___________________