Cubic Polynomial - Application

Warm-up 1

A car dealership's profit can be modeled by the function \( P(x) = x^3 + 2x^2 + 400x - 400 \), where \( x \) is the number of cars sold. How many cars will they have to sell to make $40,000 in profit?

Find the solution: \( 29.87 \), so between 29 and 30 cars

Explain two ways that your calculator could help you solve this problem.
1) Set the equation equal to zero, find the x-intercepts \( x^3 + 2x^2 + 400x - 400 = 0 \)
2) \( y_1 = x^3 + 2x^2 + 400x - 400 \) \( y_2 = 40,000 \) Find the intersection

Warm-up 2

Mars Inc. is going to make an open-top box by cutting equal squares from the four corners of a 12 inch by 18 inch sheet of cardboard and folding up the sides.

Sketch a diagram to represent this problem.

```
18-2x
 x
\( \times \)
12-2x
```

Write a polynomial to represent the area of the base of the box:

\[
\text{Area} = \text{length} \times \text{width} = (18 - 2x)(12 - 2x) = 216 - 60x + 4x^2
\]

Write a polynomial to represent the volume of the box:

\[
\text{Volume} = \text{length} \times \text{width} \times \text{height} = (4x^2 - 60x + 216)(x) = 4x^3 - 60x^2 + 216x
\]

What size square should Joe cut out to yield the maximum possible volume?

\( x = 2.35 \) in \( \text{Max Volume} = 228.16 \text{ in}^3 \)

Square would be \( 2.35 \times 2.35 \)
Cubic Polynomial - Application

**Homework/Classwork**

1. You are designing a swimming pool with a volume of 4800 ft³. The width of the pool should be 7 feet more than the depth. The length should be 32 feet more than the depth. What should the dimensions of this rectangular swimming pool be?

   \[
   \begin{align*}
   L &= D + 32 \\
   W &= D + 7 \\
   D &= D \\
   \text{Vol} &= l \cdot w \cdot h \\
   4800 &= (D + 32)(D + 7)(D) \\
   4800 &= (D^2 + 39D + 224)(D) \\
   4800 &= D^3 + 39D^2 + 224D \\
   D &= 8 \\
   W &= 15 \\
   L &= 40
   \end{align*}
   \]

2. The height of a box that Caleb is shipping is 3 inches less than the width of the box. The length is 2 inches more than twice the width. The volume of the box is 1540 in³. What are the dimensions of the box?

   \[
   \begin{align*}
   W &= W \\
   h &= W - 3 \\
   l &= 2W + 2 \\
   \text{Vol} &= W(W-3)(2W+2) = 1540 \\
   \end{align*}
   \]

3. A rectangular shipping container has a volume of 2500 cm³. The container is 4 times as wide as it is long, and 5 cm taller than it is wide. What are the dimensions of the container?

   \[
   \begin{align*}
   l &= l \\
   w &= 4l \\
   h &= 4l + 5 \\
   \text{Vol} &= (l)(4l)(4l+5) = 2500 \\
   4l^2(4l+5) &= 2500 \\
   16l^3 + 20l^2 &= 2500 \\
   l &= 5 \\
   w &= 20 \\
   h &= 25
   \end{align*}
   \]

4. Joe is going to make an open-top box by cutting equal squares from the four corners of an 11 inch by 14 inch sheet of cardboard and folding up the sides.

   Sketch a diagram to represent this problem.

   Write a polynomial to represent the area of the base of the box:

   \[
   \text{Area} = (11-2x)(14-2x) = 4x^2 - 50x + 154
   \]

   Write a polynomial to represent the volume of the box:

   \[
   \text{Volume} = (4x^2 - 50x + 154)(x) = 4x^3 - 50x^2 + 154x
   \]

   What size square should Joe cut out to yield the maximum possible volume?

   Squares that are 2.04 x 2.04 give a maximum volume of 140.03 in³