Area of Triangles

There are 3 ways to find the area of a triangle:

1) \[ Area = \frac{1}{2} \text{base} \times \text{height} \]

The problem is that we often do not know the height of the triangle.

2) If a triangle has given information Side-Angle-Side (SAS), then

\[
Area = \frac{1}{2} bcsin(A) \\
Area = \frac{1}{2} absin(C) \\
Area = \frac{1}{2} acsin(B)
\]

Where a, b, and c would the sides of the triangle and A, B, or C should be an INCLUDED angle!

3) Heron's Formula - this will work for Side-Side-Side (SSS)

First find the semiperimeter (s)…(half of the perimeter)

\[ s = \frac{1}{2}(a + b +c) \]

Then, use the semiperimeter to calculate area using the following formula.

\[ Area = \sqrt{s(s-a)(s-b)(s-c)} \]
Example 1: A surveyor designs a triangular parking lot. Two sides measure 200ft and 300ft. The angle between the sides is 65°. Find the area of the parking lot.

Example 2: A triangular building plot has two sides measuring 120ft and 75ft long. The angle included between the sides is 85°. Find the area of the building plot.

Example 3: Find the area of a triangle with sides 6, 7, and 11.

Example 4: Find the area of a triangle with sides 5, 12, 13 (what type of triangle is this one? How do you know?)

Use Heron’s: \[ \text{Use } \text{SAS: Area} = \frac{1}{2} bcsin(A) \]

Use \[ \text{Area} = \frac{1}{2} bh \]
Trigonometry and Area Practice

Find the area of each triangle using Heron’s Formula.

1. \(a = 20, b = 12, c = 28\)
2. \(a = 5, b = 12, c = 13\)
3. \(a = 11, b = 13, c = 16\)
4. \(a = 14, b = 9, c = 8\)
5. \(a = 8, b = 7, c = 3\)

Find the area of each triangle. Round your answers to the nearest tenth.

9. \(\text{6.5 m} \quad 63^\circ \quad 13 \text{ m}\)
10. \(\text{9 mi} \quad 42^\circ \quad 10 \text{ mi}\)
11. \(\text{10 km} \quad 38^\circ \quad 18 \text{ km}\)
12. \(\text{34 in.} \quad 54^\circ \quad 26 \text{ in.}\)
13. \(\text{6 mm} \quad 46^\circ \quad 4.5 \text{ mm}\)
14. \(\text{28 in.} \quad 59^\circ \quad 32 \text{ in.}\)
15. \(\text{10 cm} \quad 35^\circ \quad 19 \text{ cm}\)
16. \(\text{65}^\circ \quad 5 \text{ ft}\)
17. \(\text{4 ft} \quad 46^\circ \quad 15 \text{ m}\)