Fair Division Practice

1. Three players (one divider and two choosers) are going to divide a cake fairly using the lone divider method. The divider cuts the cake into three slices ($s_1$, $s_2$, and $s_3$). If the choosers declarations are Chooser 1: {$s_1, s_3$} and Chooser 2: {$s_3$}, which of the following is a fair division of the cake?

A Chooser 1 gets $s_2$; Chooser 2 gets $s_1$; Divider gets $s_3$.
B Chooser 1 gets $s_3$; Chooser 2 gets $s_1$; Divider gets $s_2$.
C Chooser 1 gets $s_1$; Chooser 2 gets $s_3$; Divider gets $s_2$.
D Chooser 1 gets $s_1$; Chooser 2 gets $s_2$; Divider gets $s_3$.
E None of the above

The next two questions refer to the following situation: Three players (one divider and two choosers) are going to divide a cake fairly using the lone divider method. The divider cuts the cake into three slices ($s_1$, $s_2$, and $s_3$).

2. If the choosers declarations are Chooser 1: {$s_2$} and Chooser 2: {$s_3$}, which of the following is a fair division of the cake?

A Chooser 1 gets $s_3$; Chooser 2 gets $s_2$; Divider gets $s_1$.
B Chooser 1 gets $s_1$; Chooser 2 gets $s_2$; Divider gets $s_3$.
C Chooser 1 gets $s_2$; Chooser 2 gets $s_3$; Divider gets $s_1$.
D Chooser 1 gets $s_2$; Chooser 2 gets $s_1$; Divider gets $s_3$.
E None of the above

3. Suppose the choosers value the slices as follows:

<table>
<thead>
<tr>
<th></th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chooser 1</td>
<td>30%</td>
<td>40%</td>
<td>30%</td>
</tr>
<tr>
<td>Chooser 2</td>
<td>32%</td>
<td>32%</td>
<td>35%</td>
</tr>
</tbody>
</table>

Which of the following is a fair division of the cake?
A Chooser 1 gets $s_2$; Chooser 2 gets $s_1$; Divider gets $s_3$.
B Chooser 1 gets $s_3$; Chooser 2 gets $s_2$; Divider gets $s_1$.
C Chooser 1 gets $s_2$; Chooser 2 gets $s_3$; Divider gets $s_1$.
D Chooser 1 gets $s_1$; Chooser 2 gets $s_2$; Divider gets $s_3$.
E None of the above

31. An estate consisting of a car, a boat, a house and a collection of rare books must be divided fairly among five heirs. This type of problem is called
A a discrete fair division problem
B a mixed fair division problem
C a continuous fair division problem
D the method of sealed bids
E None of the above
Fair Division Practice

The next four questions refer to the following example: Four heirs (A, B, C, and D) must divide fairly an estate consisting of three items - a house, a cabin and a boat - using the method of sealed bids. The players' bids (in dollars) are:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>180,000</td>
<td>200,000</td>
<td>190,000</td>
<td>185,000</td>
</tr>
<tr>
<td>Cabin</td>
<td>60,000</td>
<td>50,000</td>
<td>40,000</td>
<td>55,000</td>
</tr>
<tr>
<td>Boat</td>
<td>16,000</td>
<td>12,000</td>
<td>18,000</td>
<td>10,000</td>
</tr>
</tbody>
</table>

26. The original fair share of player B is worth
   A $64,000
   B $62,000
   C $62,500
   D $65,500
   E None of the above

27. In the initial allocation, player B
   A gets the cabin and an additional $15,500 from the estate.
   B gets $65,500 cash
   C gets the house and pays the estate $134,500.
   D gets the boat and an additional $53,500 from the estate.
   E None of the above

28. After the initial allocation to each player is made there is a surplus of
   A $0
   B $6,000
   C $12,000
   D $24,000
   E None of the above

29. After all is said and done, the final allocation to player B is:
   A the house minus $134,500 in cash.
   B $65,500 in cash.
   C the house minus $128,500 in cash
   D the house plus $6,000 in cash.
   E None of the above

30. Four players (A, B, C, and D) agree to divide the 15 items shown below by lining them up in order and using the method of markers. The player's bids are as indicated.

List the items that each player gets below:

A:  
B:  
C:  
D:  
Leftovers:
32. Joe and Bill want to divide a cake using the divider-chooser method. They draw straws, and it is determined that Bill will be the divider and Joe the chooser. Assuming that each plays the game correctly, which of the following statements cannot be true? (Select "None of the above" if the other four statements are true.)

A) Joe believes that his share is worth 60% of the cake; Bill believes that his share is worth 50% of the cake.
B) Bill believes that Joe's share is worth 50% of the cake; Joe believes that his share is worth 60% of the cake.
C) Joe believes that his share is worth 50% of the cake; Bill believes that his share is worth 50% of the cake.
D) Bill believes that his share is worth 60% of the cake; Joe believes that his share is worth 50% of the cake.

34. Sue and Tom are getting a divorce. Except for the house they own very little of value so they agree to divide the house fairly using the method of sealed bids. Sue bids 100,000 and Tom bids 90,000. After all is said and done, the final outcome is
A) Sue gets the house and pays Tom 50,000
B) Sue gets the house and pays Tom 47,500
C) Sue gets the house and pays Tom 45,000
D) Sue gets the house and pays Tom 55,000
E) None of the above

Four players (A, B, C, D) agree to divide the 12 items shown in Figure 3.5 using the method of markers. The player’s bids are as indicated.

44. Item 4 goes to Player ______
   Item 10 goes to Player ______
   Item 7 goes to Player ______

45. Which player receives the largest number of items? ______
Fair Division Practice
The next three questions refer to the following situation:

Aaron and Brittney want to divide fairly the chocolate/banana cake shown in the figure using the divider chooser method. The total cost of the cake was $6.00. Aaron values chocolate five times as much as he values banana, while Brittney values banana twice as much as she values chocolate.

35. In Aaron’s eyes, the piece shown in the figure below is worth
A $2.00  
B $3.00  
C $2.50  
D $3.50  
E None of the above

36. In Aaron’s eyes, the piece shown in the figure is worth:  
(the angles are 45 degrees and 90 degrees)  
A $1.50  
B $1.75  
C $1.25  
D $1.00  
E None of the above

37. In Brittney’s eyes, the piece shown in the figure is worth  
(the angles are 45 degrees and 90 degrees)  
A $2.00  
B $2.50  
C $3.00  
D $3.50  
E None of the above
39. Three players (one divider and two choosers) are going to divide a cake fairly using the *lone divider method*. The divider cuts the cake into three slices ($s_1$, $s_2$, and $s_3$). Suppose the choosers value the slices as follows:

\[
\begin{array}{ccc}
\text{Chooser 1} & \text{Chooser 2} \\
38\% & 29\% \\
32\% & 29\% \\
30\% & 41\%
\end{array}
\]

- **A** Chooser 1 gets $s_2$; Chooser 2 gets $s_3$; Divider gets $s_1$.
- **B** Chooser 1 gets $s_2$; Chooser 2 gets $s_1$; Divider gets $s_3$.
- **C** Chooser 1 gets $s_1$; Chooser 2 gets $s_3$; Divider gets $s_2$.
- **D** Chooser 1 gets $s_3$; Chooser 2 gets $s_2$; Divider gets $s_1$.
- **E** None of the above

Four heirs (A, B, C, and D) must divide fairly an estate consisting of three items – a house, a parcel of land, and a diamond ring – using the method of sealed bids. The players’ bids (in dollars) are:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>140,000</td>
<td>155,000</td>
<td>160,000</td>
<td>148,000</td>
</tr>
<tr>
<td>Land</td>
<td>80,000</td>
<td>90,000</td>
<td>100,000</td>
<td>75,000</td>
</tr>
<tr>
<td>Ring</td>
<td>40,000</td>
<td>35,000</td>
<td>36,000</td>
<td>25,000</td>
</tr>
</tbody>
</table>

40. The original fair share of player A is worth
- **A** $260,000.
- **B** $65,000.
- **C** $86,667.
- **D** $130,000.
- **E** None of the above

41. In the initial allocation, player A
- **A** gets the ring and pays the estate $25,000.
- **B** gets the house and pays the estate $75,000.
- **C** gets the land and pays the estate $15,000.
- **D** gets the ring and an additional $25,000 from the estate.
- **E** None of the above

42. After the initial allocation to each player is made there is a surplus of
- **A** $186,000.
- **B** $0.
- **C** $101,000.
- **D** $29,000.
- **E** None of the above

43. After all is said and done, the final allocation to player B is:
- **A** $99,000 in cash.
- **B** $77,250 in cash.
- **C** the ring and $32,250 in cash.
- **D** $70,000 in cash.
- **E** None of the above