Dividing Polynomials

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Dividing by a Monomial

If the divisor only has one term, split the polynomial up into a fraction for each term.

\[
\frac{18x^4 - 24x^3 + 6x^2 - 12x}{6x} = \frac{18x^4}{6x} - \frac{24x^3}{6x} + \frac{6x^2}{6x} - \frac{12x}{6x}
\]

Now reduce each fraction.

\[
\frac{18x^4}{6x} - \frac{24x^3}{6x} + \frac{6x^2}{6x} - \frac{12x}{6x} = 3x^3 - 4x^2 + x - 2
\]
Long Division

If the divisor has more than one term, perform long division. You do the same steps with polynomial division as with integers. Let's do two problems, one with integers you know how to do and one with polynomials and copy the steps.

D - Divide
M - Multiply
S - Subtract
B - Bring Down
R - repeat

\[ \begin{array}{c}
32 & \overline{1698} \\
-64 & \underline{64} \\
58 & \underline{58} \\
-32 & \underline{-32} \\
26 & \\
\end{array} \]
Using Long Division to Divide Polynomials

Divide by using long division:

\[ 277 \div 12 \]
**Long Division**

If the divisor has more than one term, perform long division. You do the same steps with polynomial division as with integers. Let's do two problems, one with integers you know how to do and one with polynomials and copy the steps.

1. Divide
2. Multiply
3. Subtract
4. Bring Down

\[
\begin{align*}
\text{Divisor} & : x + 11 \\
\text{Dividend} & : x^2 - 3x^2 + 8x - 5 \\
\end{align*}
\]

\[
\begin{align*}
\text{Quotient} & : x \\
\text{Remainder} & : 11 \times -5 \\
\text{Answer} & : x + 11 \div x - 3 \text{ R } 28
\end{align*}
\]
Using Long Division to Divide Polynomials

Divide by using long division:

\[
\frac{15x^2 + 8x - 12}{3x + 1}\]

Steps:
1. Divide the first term of the dividend by the first term of the divisor: 
   \[\frac{15x^2}{3x} = 5x\]
2. Multiply the divisor by the result: 
   \[(3x + 1) \cdot 5x = 15x^2 + 5x\]
3. Subtract: 
   \[15x^2 + 8x - (15x^2 + 5x) = 3x - 12\]
4. Bring down the next term: 
   \[3x - 12\]
5. Repeat the process:
   - Divide: \[\frac{3x}{3x} = 1\]
   - Multiply: \[(3x + 1) \cdot 1 = 3x + 1\]
   - Subtract: \[3x - (3x + 1) = -1\]

The quotient is \(5x + 1\) and the remainder is \(-13\).
Using Long Division to Divide Polynomials

Divide by using long division:

\[ (2 + 5 \; -28) \div (-3) \]

1. Divide
2. Multiply
3. Subtract
4. Bring Down

\[
\begin{align*}
\frac{x^2}{x^1} &= x \\
\frac{x^2}{x^2} &= \frac{8}{3} \\
\frac{8x - 28}{-8x + 24} &\rightarrow -4 \\
\text{remainder} &\rightarrow -4
\end{align*}
\]
Let's Try Another One

If any powers of terms are missing you should write them in with zeros in front to keep all of your columns straight.

\[
\frac{y^2 + 8}{y + 2}
\]
Dividing Polynomials Notes

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Using Long Division to Divide Polynomials

Divide by using long division:

\[
(4x^3 + 3x^2 + 2x + 10) \div (-2)
\]

\[
\begin{array}{c|ccccc}
& 4x^2 & +11x & +22 \\
\hline
x-2 & 4x^3 & +3x^2 & +0x & +10 \\
\hline
& -4x^3 & -8x^2 & \\
& -4x^3 & -8x^2 & +11x^2 & +22x \\
\hline
& & 11x^2 & +0x & +10 \\
& & -11x^2 & -22x & \\
\hline
& & & 22x & +10 \\
& & & -22x & -44 & \Rightarrow \boxed{54}
\end{array}
\]

\[
\boxed{2} \begin{array}{c|ccccc}
& 4 & 3 & 0 & 10 \\
\hline
\downarrow & 8 & 22 & 44 \\
\hline
& 4 & 11 & 22 & 54
\end{array}
\]
Determine whether each binomial is a factor of

\[ x^3 + 4x^2 + x - 6 \]

\[ x^2 + 3x - 2 - \frac{4}{x+1} \]

1. Divide
2. Multiply
3. Subtract
4. Bring Down

\[ (x+1)(x^2 + 2x - 3) \]

\[ x^2 + 2x - 3 \]

\[ x^3 + 4x^2 + x - 6 \]

\[ -x^3 - 2x^2 \]

\[ 3x^2 + x \]

\[ -3x^2 + 3x \]

\[ -2x - 6 \]

\[ x^2 + 2x + 2 \]

remainder \(\Rightarrow -4\)
SYNTHETIC DIVISION

Divide: \( x^3 - 2x^2 - 8x - 35 \) by \((x - 5)\)

**Steps:**
1. Write the known zero in the house.
2. List out the coefficients.
3. Bring down the 1st coefficient.
4. Multiply the 1st coefficient by the house number.
5. Write the product under the 2nd coefficient.
6. Add down.
7. Repeat.
8. Use final numbers to write polynomial.
9. Use the Quadratic Formula to find the other zeros.

**Zero is 5**

**Grab the coefficients**
\( x^3 - 2x^2 - 8x - 35 \)

**Multiply**

**Answer:** \( x^2 + 3x + 7 \)

\( x - 5 = 0 \)

\( x = 5 \)

**Cubic**
\( x^3 - 2x^2 - 8x - 35 \div (x - 5) \)

\( \boxed{1} \)

\( \boxed{1} \)

\( \boxed{5} \)

\( \boxed{5} \)

\( \boxed{3} \)

\( \boxed{7} \)

Quadratic
\( x^2 + 3x + 7 \)

Remainder
0
Dividing Polynomials Notes

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**Synthetic Division**

There is a shortcut for long division as long as the divisor is $x - k$ where $k$ is some number (Can't have any powers on $x$).

\[ (x-10) \quad (x+3) \]

\[ \begin{array}{c|ccc}
\text{Root} & 1 & 6 & 8 & -2 \\
\hline
-3 & \downarrow & 3 & -9 & 3 \\
\hline
1 & 3 & -1 & \uparrow & \text{remainder} \\
\end{array} \]
Let's try another Synthetic Division

\[
\begin{array}{c|cccc}
0 & -4 & 0 & 6 \\
4 & 4 & 16 & 48 & 192 \\
\hline
4 & 12 & 48 & 198 \\
\end{array}
\]

\[
x^3 + 4x^2 + 12x + 48 + \frac{198}{x-4}
\]

Root: 4

Root: 4

Quadratic:

\[
x^4 - 4x^2 + 6
\]

\[
x - 4
\]
$\frac{x^3 + 3x^2 - x - 3}{x - 1}$

1. Divide: $x^2 + 4x + 3$
2. Multiply: $x^2 + 4x + 3$
3. Subtract: $-x^2 - x - 3$
4. Bring down: $1$

$\frac{x^3}{x} = x^2$

$\frac{4x^2}{x} = 4x$

$x^2 + 4x + 3$

$\frac{3x}{1} = 3x$

$-3x - 3$

$0$
Let's try a problem where we check to see if the given expression is one of its factors.

\[ x + 2 = 0 \quad \Rightarrow \quad x = -2 \]

Check if \( 4x^3 + 8x^2 - 25x - 50 \) is a factor of \( x + 2 \).

\[
\begin{array}{cccc}
-2 & | & 4 & 8 & -25 & -50 \\
& -8 & 0 & 50 & \hline
& 4 & 0 & -25 & 0
\end{array}
\]

Remainder: 0

Yes!!

\( x + 2 \) is a factor of \( 4x^3 + 8x^2 - 25x - 50 \).
Determine which binomial is a factor of \(-2x^3 + 5x^2 + 16x - 16\).

a.  \(x + 16\)  

b.  \(x + 4\)  

c.  \(x - 16\)  

d.  \(x - 4\)

\[\begin{array}{cccc}
-16 & -2 & 5 & 16 & -16 \\
\downarrow & 32 & -592 & 9216 \\
-2 & 37 & -576 & 9200 \\
\end{array}\]

\[\begin{array}{cccc}
-4 & -2 & 5 & 16 & -16 \\
\downarrow & 8 & -52 & 144 \\
-2 & 13 & -36 & 128 \\
\end{array}\]

\[\begin{array}{cccc}
-16 & -2 & 5 & 16 & -16 \\
\downarrow & -32 & -432 & -6656 \\
-2 & -27 & -4161 \\
\end{array}\]

\[\begin{array}{cccc}
4 & -2 & 5 & 16 & -16 \\
\downarrow & 8 & -12 & 16 \\
-2 & -3 & 4 & 0 \\
\end{array}\]